Theory of Tunneling in the Exciton Condensate of Bilayer Quantum Hall Systems

K. Park
Condensed Matter Theory Center, Department of Physics,
University of Maryland, College Park, MD 20742-4111
(Dated: February 2, 2008)

We develop a theory of interlayer tunneling in the exciton condensate of bilayer quantum Hall systems, which predicts strongly enhanced, but finite, tunneling conductance peaks near zero bias even at zero temperature. It is emphasized that, though this strongly enhanced tunneling originates from spontaneous interlayer phase coherence, it is fundamentally not the Josephson effect. Because of strong interlayer correlation, the bilayer system behaves as a single system so that conventional tunneling theories treating two layers as independent systems are not applicable. Based on our theory, we compute the height of conductance peak as a function of interlayer distance, which is in good agreement with experiment.

PACS numbers: 73.43.-f, 73.21.-b

When Spielman *et al.* [1] observed strongly enhanced interlayer conductance peaks near zero bias in bilayer quantum Hall systems at total filling factor $\nu_T = 1$, they not only renewed our interest in the bilayer quantum Hall effect [2], but also attracted intense interest from the general perspective of strongly correlated physics. It was because, in addition to its many-body origin, the bilayer quantum Hall effect bears a rather precise analogy to superconductivity; the ground state of bilayer quantum Hall effect at interlayer distance $d/l_B \ll 1$ ($l_B = \sqrt{\hbar c/eB}$) maps onto the BCS wavefunction of an exciton condensate of particle-hole pairs formed across the interlayer barrier. In fact, Bose-Einstein condensation of excitons in semiconductors has been sought after for decades. In particular, there have been fascinating recent experiments on the possible condensation of optically generated indirect excitons [3], for which, however, there is not yet conclusive evidence. On the other hand, it is generally accepted that the strongly enhanced conductance peak in the quantum Hall regime is a direct indication of macroscopic phase coherence.

To be concrete regarding the mapping between the superconductivity and bilayer quantum Hall effect, let us write the exact ground state wavefunction at $d/l_B = 0$, *i.e.* the Halperin's (1,1,1) state [4] (which is adiabatically connected to the ground states at sufficiently small, but finite d/l_B):

$$|\psi_{111}\rangle = \prod_m (c^\dagger_{m\uparrow} + c^\dagger_{m\downarrow})|0\rangle, \quad (1)$$

where m is a momentum index in the lowest Landau level and the pseudospin representation is used: $\uparrow$ ($\downarrow$) indicates the top (bottom) layer. Note that Eq.(1) describes the full wavefunction including both orbital and layer degree of freedom [5]. Since Eq.(1) has a structure isomorphic to the BCS wavefunction, it is clear that the bilayer quantum Hall state should have a phase coherence between states with different interlayer number difference in analogy with phase coherence between different number eigenstates in superconductivity, which is the origin of the Josephson effect. Naturally, this similarity led previous authors [6, 7] to predict the Josephson effect in bilayer quantum Hall systems. The strongly enhanced conductance observed by Spielman *et al.*, therefore, seemed to be exactly the experimental verification needed. However, there are key properties of the conductance peak indicating that this phenomenon is not the conventional Josephson effect: most notably, saturation of height as well as width to finite values in the limit of zero temperature [8].

This apparent discrepancy gave rise to two groups of thought. In one group, the enhanced conductance is still regarded as DC Josephson effect, but its height is reduced by complicated disorder-induced fluctuations [9, 10, 11, 12]. On the other hand, others [13] argued that there is no exact analog of Josephson effect in interlayer tunneling experiments because the bilayer system as a whole is a single superfluid, not a set of two superfliud systems. While we agree with the latter viewpoint that the enhanced interlayer tunneling conductance is not the analog of Josephson effect, we show below that strong interlayer correlation requires a fundamentally new starting point different from all of above theories in order to construct a self-consistent theory of interlayer tunneling in quantum Hall regime.

As mentioned previously, the bilayer quantum Hall system is a single superfluid system. So, it is impossible to induce a chemical potential gradient between the two layers without destroying interlayer phase coherence, in which case the interlayer current becomes a normal current, not supercurrent. It is important to distinguish between the chemical potential gradient and applied interlayer bias voltage because, even when the bias voltage is applied, bilayer systems will immediately reach an equilibrium by creating charge imbalance in order to compensate the relative voltage difference and therefore there is no chemical potential gradient. Though this point seems straightforward, it has been completely overlooked by all previous theories which, regardless of their viewpoint

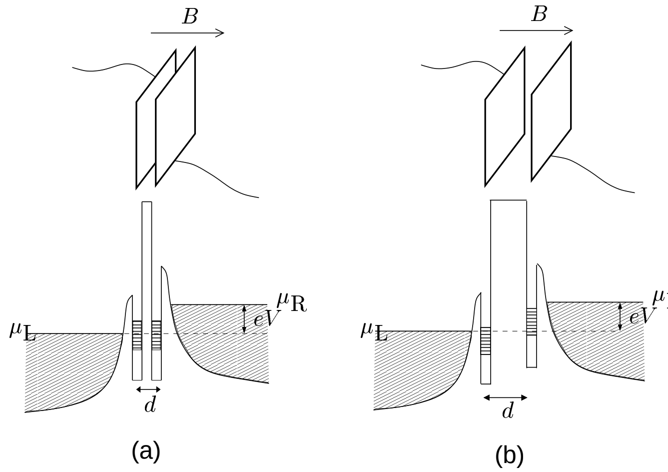


FIG. 1: Schematic diagram of tunneling measurement in bilayer quantum Hall systems. Note that there is no chemical potential gradient between layers when the ground state of bilayer system becomes a single exciton condensate at small interlayer distance d, as depicted in (a). A consistent theory of interlayer tunneling, therefore, should inevitably take external leads into consideration. On the other hand, when d is sufficiently large as shown in (b), two layers behave as independent systems, and interlayer coherence is lost.

regarding the analogy with Josephson effect, began by implicitly making a self-contradictory assumption that there is strong interlayer correlation due to the Coulomb interaction but two layers can be treated independently by having a finite chemical potential gradient. In fact, if one can induce a finite chemical potential gradient while maintaining interlayer phase coherence, there would be a very interesting experimental consequence: oscillating tunneling current whose frequency is proportional to the applied bias voltage. However, no oscillating current has been observed in experiments.

Now, if there is no interlayer chemical potential gradient, there is no electromotive force within bilayer system and any current should be induced from outside. It is, therefore, necessary to take into account external leads, as schematically shown in Fig.1. This, of course, makes any quantitative prediction dependent on the way in which bilayer systems are connected to external leads. However, it is still possible to make a quantitative prediction on essential aspects of coherent interlayer tunneling. In particular, we will compute the dependence of tunneling conductance peak height on interlayer distance d/l_B. Also, we will show that the width is finite even at zero temperature, and it is controlled ultimately by extremely small, but finite single-particle interlayer tunneling gap $\Delta_{\rm SAS}$.

Let us begin our quantitative analysis by writing the total Hamiltonian including the Hamiltonian for Coulomb interaction between electrons in bilayer system H_0, the Hamiltonian describing the left and right lead, H_L and H_R respectively, and tunneling between leads and the bilayer system H':

$$H = H_0 + H' + H_R + H_L, \tag{2}$$

$$\frac{H_0}{e^2/\epsilon l_B} = \mathcal{P}_{LLL}\Big(\sum_{i,j\in\uparrow}\frac{1}{r_{ij}} + \sum_{k,l\in\downarrow}\frac{1}{r_{kl}} + \sum_{i\in\uparrow,k\in\downarrow}\frac{1}{\sqrt{r_{ik}^2+(d/l_B)^2}}\Big)\mathcal{P}_{LLL}, \tag{3}$$

$$H' = \sum_{k,m} T_{R\uparrow}(k,m)[c_R^\dagger(k)c_{m\uparrow} + \text{h.c.}] + \sum_{p,m'} T_{L\downarrow}(p,m')[c_L^\dagger(p)c_{m'\downarrow} + \text{h.c.}], \tag{4}$$

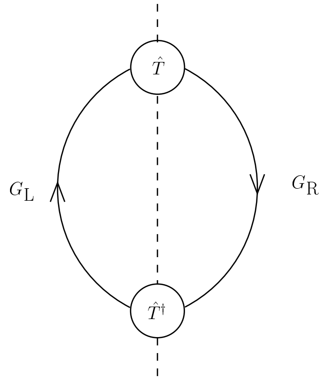
where, as before, the pseudospin representation is used, and $\mathcal{P}_{LLL}$ is the lowest Landau level projection operator. $T_{R\uparrow}(k,m)$ is the tunneling amplitude between the state with momentum k in the right lead, and the state with m in the top layer of bilayer system. $T_{L\downarrow}(p,m')$ is similarly defined. H_R and H_L describe electrons in external leads as normal Fermi liquids. It is now very important to note that H does not have any interlayer tunneling term within the bilayer system. It is because we are interested in the spontaneous interlayer coherence which occurs in the limit of zero interlayer tunneling gap: $\Delta_{\rm SAS}/(e^2/\epsilon l_B) \to 0$. As will be shown later, this spontaneous interlayer coherence is due to the many-body effect of Coulomb interaction in H_0, and it creates a non-zero current from one layer to the other even in the limit of zero interlayer tunneling gap (of course, in unbiased equilibrium, the net current is zero since two opposite currents cancel each other).

Since there is no direct process of transporting electrons from one lead through the bilayer system to the other lead, one has to consider second order tunneling processes:

$$H'_T = H'\frac{1}{E_g - H_0 - H_R - H_L}H', \tag{5}$$

where E_g is the ground state energy of $H_0 + H_R + H_L$. By adding an electron to the top layer and removing another from the bottom layer, H'_T describes tunneling processes through the bilayer system. Now, because the bilayer quantum Hall state is incompressible at sufficiently small d/l_B, adding or removing electrons costs a finite energy which is equal to the Coulomb self-energy of quasiparticles, Δ_C [14]. We will compute Δ_C as a function of d/l_B later by using exact diagonalization. It is, however, sufficient at this stage to know that Δ_C is independent of momentum m in the lowest Landau level. So one can just replace $H_0 + H_R + H_L - E_g$ by Δ_C. Remember that there is no energy cost in taking electrons from external leads because normal Fermi liquids are compressible.

Now, we assume that the tunneling amplitudes $T_{R\uparrow}(k,m)$ and $T_{L\downarrow}(p,m')$ are more or less independent



$$\hat{T} = \frac{1}{\Delta_C}\Sigma_m\, T_{RL}(m) c^\dagger_{m\uparrow} c_{m\downarrow}$$

FIG. 2: Feynman diagram of interlayer tunneling in bilayer quantum Hall systems. The vertex operator $\hat{T}$ contains all of many-body effects of an exciton condensate. T_{RL} is the tunneling amplitude and Δ_C is the Coulomb self-energy of quasiparticle.

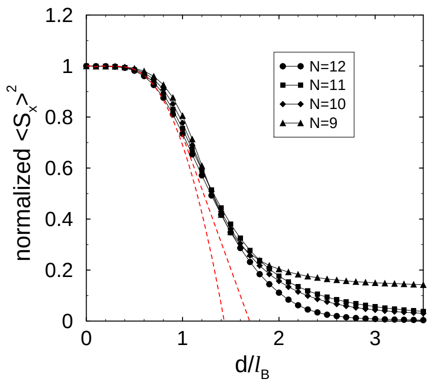


FIG. 3: Normalized expectation value of condensate order parameter $\langle S_x\rangle^2$. Dashed lines indicate the upper and lower bound for an estimate of the thermodynamic limit of $\langle S_x\rangle^2$ as a function of d/l_B. N is the total number of electrons.

of momenta k and p, which is a common practice in tunneling theories when studying tunneling processes only within a narrow region of energy near Fermi surface. Keeping only terms of H'_T relevant for transporting electrons from one lead to the other, we arrive at the following tunneling Hamiltonian:

$$H_T = \sum_{k,p}\left[c^\dagger_R(k)c_L(p)\hat{T}^\dagger + c^\dagger_L(p)c_R(k)\hat{T}\right], \qquad (6)$$

where

$$\hat{T} = \frac{1}{\Delta_C}\sum_m T_{RL}(m)c^\dagger_{m\uparrow}c_{m\downarrow} \qquad (7)$$

and $T_{RL}(m) = T_{R\uparrow}(k_F,m)T_{L\downarrow}(k_F,m)$ [15]. Based on H_T, the tunneling current operator $\hat{J}$ is given as follows:

$$\hat{J} = ei\sum_{k,p}\left[c^\dagger_R(k)c_L(p)\hat{T}^\dagger - c^\dagger_L(p)c_R(k)\hat{T}\right]. \qquad (8)$$

We now compute the expectation value of current operator via a conventional first-order S-matrix expansion:

$$I(t) = -i\int_{-\infty}^{t} dt'\langle[\hat{J}(t), H_T(t')]\rangle. \qquad (9)$$

The new aspect of our tunneling theory is the vertex operator $\hat{T}$ which contains all of many-body effects of the exciton condensate. Eq.(9) can be evaluated further using the Feynman diagram depicted in Fig.2:

$$\begin{aligned} I &= 2e|\langle\hat{T}\rangle|^2\sum_{k,p}\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}A_R(k,\varepsilon)A_L(p,\varepsilon+eV) \\ &\times\ [f(\varepsilon)-f(\varepsilon+eV)] \\ &= 4\pi e^2 D_R D_L|\langle\hat{T}\rangle|^2 V \end{aligned} \qquad (10)$$

where A_R (A_L) is the spectral function of the right (left) lead, $f(\varepsilon)$ is the usual Fermi-Dirac distribution function, and D_R (D_L) is the density of states at the Fermi surface of right (left) lead. It is clear from Eq.(10) that there is no DC Josephson effect because the conductance G ($\equiv dI/dV \propto |\langle\hat{T}\rangle|^2$) is finite. However, the interlayer tunneling current is zero unless there is a phase coherence: $\langle\hat{T}\rangle \neq 0$. Remember that $\langle\hat{T}\rangle$ measures a phase coherence between states with different values of interlayer number difference, $N_{\rm rel}$, because $\hat{T} \propto c^\dagger_{m\uparrow}c_{m\downarrow}$ and therefore changes $N_{\rm rel}$ by two. So, unless the ground state is a coherent linear combination of states with various $N_{\rm rel}$, $\langle\hat{T}\rangle$ is zero, and so is the tunneling current. As mentioned before, this is similar to the phase coherence between different number eigenstates in superconductivity, which is responsible for the Josephson effect. In this sense, interlayer tunneling conductance is related to the Josephson effect. However, we emphasize that the conductance should be finite even at zero temperature and there is no direct analogy with the Josephson effect. We now compute the interlayer tunneling conductance as a function of d/l_B. In particular, we will be interested in normalized conductance since the absolute scale of conductance is sensitive to sample-specific details such as D_R, D_L and $T_{\rm RL}$.

In essence, we compute $|\langle\hat{T}\rangle|^2$ which can be further reduced as follows:

$$|\langle\hat{T}\rangle|^2 = \frac{\langle S_x\rangle^2}{\Delta_C^2}\left|\frac{1}{N}\sum_m T_{\rm RL}(m)\right|^2, \qquad (11)$$

where N is the total number of electrons, and we have used the fact that $\langle c^\dagger_{m\uparrow}c_{m\downarrow}\rangle$ is independent of m and is equal to $\langle S_x\rangle/N$. S_x [$= \sum_m(c^\dagger_{m\uparrow}c_{m\downarrow} + c^\dagger_{m\downarrow}c_{m\uparrow})/2$] is the order parameter of exciton condensation, and it can also be interpreted as the pseudospin magnetization in the x direction. Since $\sum_m T_{\rm RL}(m)/N$ does not depend on d/l_B, the interlayer distance dependence of conductance

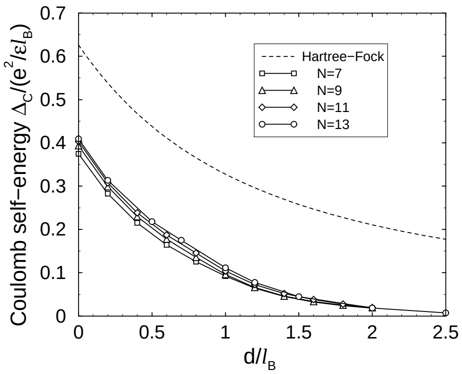


FIG. 4: Coulomb self-energy of a quasiparticle.

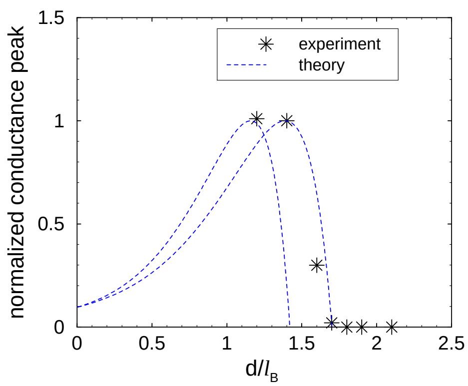


FIG. 5: Normalized interlayer tunneling conductance peak as a function of interlayer distance in comparison with experimental data from Ref.[1]. We define the normalized conductance as conductance divided by its maximum value as a function of d/l_B. Two theoretical curves are obtained from the upper and lower bound of thermodynamic estimate in Fig.3.

is solely determined by $\langle S_x\rangle^2/\Delta_C^2$.

In Fig.3 we plot $\langle S_x\rangle^2$ as a function of d/l_B which is computed via exact diagonalization of finite systems with various particle numbers in torus geometry. When computing $\langle S_x\rangle$ in finite systems, it is very important to take into account fundamental fluctuations in $N_{\rm rel}$; the true ground state is a coherent, linear combination of states with various $N_{\rm rel}$ [16, 17]. Though estimating the accurate thermodynamic limit of $\langle S_x\rangle^2$ is difficult, it is reasonable to argue that the true thermodynamic limit lies between two dashed lines in Fig.3.

Fig.4 plots the Coulomb self-energy of a quasiparticle, $\Delta_C/(e^2/\epsilon l_B)$, as a function of d/l_B which is determined in exact diagonalization studies by computing the energy gap of particle-hole-pair excitation with the largest momentum and taking half of its value. For comparison, we also plot the self-energy in the Hartree-Fock approximation [18] which tends to overestimate Δ_C.

Finally, in Fig.5 we compare our estimate of normalized interlayer tunneling conductance near zero bias, *i.e.* $\langle S_x\rangle^2/\Delta_C^2$, with experimental data of Spielman *et al.* [1]. We define the normalized conductance as conductance divided by its maximum value as a function of d/l_B. Two dashed lines in Fig.5 correspond to the upper and lower bound of estimated thermodynamic limits of $\langle S_x\rangle^2$ in Fig.3. Considering simplifications used in our theory such as omission of finite thickness effect, we find our theory to be in good agreement with experiments. In addition to further comparison with experiments in the regime $d/l_B \gtrsim 1.2$, it will be very interesting to see whether our prediction of decrease of conductance peak for $d/l_B \lesssim 1.2$ is consistent with future experiments. Remember that decrease in conductance peak at small d/l_B is due to increase in energy gap to put electrons into bilayer systems while the pseudospin magnetization is saturated. We would like to emphasize that, once normalized, our theoretical estimate of conductance peak does not have any fitting parameter.

We have shown by means of Eq.(10) and (11) that in exciton condensate the interlayer tunneling conductance at small bias is finite, but strongly enhanced. However, we did not show why the conductance should be sharply peaked near zero bias, which we will explain now. Once the interlayer current is driven by an external electromotive force, it should physically flow through the bilayer system since otherwise there is no steady state. Exciton condensates accomplish this by adjusting their interlayer phase difference ϕ to sustain the externally driven current, which is again easy to understand in terms of the ground state wavefunction at $d/l_B \to 0$:

$$|\psi_{111}(\phi)\rangle = \prod_m (c^\dagger_{m\uparrow} + e^{i\phi} c^\dagger_{m\downarrow})|0\rangle, \qquad (12)$$

which carries a net internal current within bilayer system equal to $e\Delta_{\rm SAS}\frac{N}{2}\sin\phi$ [19]. Then, there should be a critical current at $\phi = \pi/2$ which is the maximum current allowed without breaking phase coherence. Therefore, for sufficiently large voltage bias, coherent interlayer currents should be cut off and become constant as a function of bias voltage, once they reach the critical value controlled by single-particle interlayer tunneling gap $\Delta_{\rm SAS}$. The conductance associated with coherent tunneling, therefore, should be zero after the critical voltage and is strongly enhanced only near zero bias. Consequently, the width of conductance peak is proportional to very small, but finite $\Delta_{\rm SAS}$, while the proportionality constant strongly depends sample-specific details such as the density of states of leads. It is, however, encouraging to find that typical width of conductance peak ($\sim 10-100\mu eV$) is roughly in the same order as $\Delta_{\rm SAS}$ [1, 8]. The above argument is valid for general d/l_B when there is phase coherence.

Until now, we have studied the interlayer tunneling conductance in a single bilayer system, which, we showed, is not the exact analog of Josephson effect. We now conclude by proposing a much more direct analog with the

Josephson effect. Consider a pair of bilayer systems, say A and B (four layers altogether), separated by a lateral tunneling barrier. Then, put an interlayer current through the top and bottom layer of, say, bilayer system A, in which way a non-zero interlayer phase difference is induced in bilayer system A while the system B has none. We predict then that there will be two counterflowing currents: one between two, top layers of system A and B, and the other between bottom layers. The net current will be zero, but it may be possible to measure these two currents individually.

This work was supported by ARDA.

[1] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **84**, 5808 (2000).

[2] S. Q. Murphy, J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **72**, 728 (1994).

[3] L. V. Butov *et al.*, Nature **418**, 751 (2002); D. Snoke *et al.*, Nature **418**, 754 (2002); C. W. Lai *et al.*, Science **303**, 503 (2004).

[4] B. I. Halperin, Helv. Phys. Acta **56** 75 (1983).

[5] K. Yang *et al.*, Phys. Rev. B **54**, 11644 (1996).

[6] X. G. Wen and A. Zee, Phys. Rev. Lett. **69**, 1811 (1992).

[7] Z. F. Ezawa and A. Iwazaki, Phys. Rev. B **47**, 7295 (1993).

[8] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **87**, 036803 (2001).

[9] A. Stern, S. Das Sarma, M. P. A. Fisher, and S. M. Girvin, Phys. Rev. Lett. **84**, 139 (2000).

[10] A. Stern, S. M. Girvin, A. H. MacDonald, and N. Ma, Phys. Rev. Lett. **86**, 1829 (2001).

[11] L. Balents and L. Radzihovsky, Phys. Rev. Lett. **86**, 1825 (2001).

[12] M. Fogler and F. Wilczek, Phys. Rev. Lett. **86**, 1833 (2001).

[13] Y. N. Joglekar and A. H. MacDonald, Phys. Rev. Lett. **87**, 196802 (2001).

[14] Bilayer quantum Hall systems have an edge excitation which requires an infinitesimally small energy to be created. However, the edge excitation does not have any contribution to coherent interlayer tunneling since it is not a part of coherent exciton condensate and its contribution to interlayer tunneling is only through single-particle tunneling which is of the order of extremely small single-particle tunneling gap $\Delta_{\rm SAS}$.

[15] In general, the vertex operator $\hat{T}$ can have contributions from terms $c^\dagger_{m\uparrow} c_{m'\downarrow}$ with $m \neq m'$. However, it has been shown [16] that the ground state at sufficiently small d/l_B has very little contribution due to $\langle c^\dagger_{m\uparrow} c_{m'\downarrow} \rangle$ when $m \neq m'$.

[16] K. Park, Phys. Rev. B **69**, 045319 (2004).

[17] K. Park, V. W. Scarola, and S. Das Sarma, Phys. Rev. Lett. **91**, 026804 (2003).

[18] H. A. Fertig, Phys. Rev. B **40**, 1087 (1989).

[19] This can be shown by computing the expectation value of internal current operator within bilayer systems: $\hat{J}_{\rm inter} = ie\Delta_{\rm SAS} \sum_m (c^\dagger_{m\uparrow} c_{m\downarrow} - c^\dagger_{m\downarrow} c_{m\uparrow})/2$ which is basically the pseudospin component in the y direction similar to S_x.